

HIGH SCHOOL GEOMETRY CHEAT SHEET

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1 BASICS AND ANGLES

Notation

Term	Notation	Meaning
Point	A	A location
Line	line AB	Extends forever in both directions
Ray	ray AB	Starts at A, passes through B, goes on forever
Segment	AB	Part of a line with two endpoints
Angle	$\angle ABC$	Vertex at B

Angle types

Type	Measure
Acute	Less than 90°
Right	Exactly 90°
Obtuse	Between 90° and 180°
Straight	Exactly 180°
Reflex	Between 180° and 360°

Angle pairs

Pair	Rule
Complementary	Add up to 90°
Supplementary	Add up to 180°
Linear pair	Adjacent and supplementary
Vertical	Opposite each other, equal

Parallel lines cut by a transversal

Angle pair	Relationship
Corresponding	Equal
Alternate interior	Equal
Alternate exterior	Equal
Co-interior (same-side interior)	Supplementary (add to 180°)

Common mistake: Co-interior angles add to 180° ; they are **not** equal. These rules work only when the lines are parallel.

2 TRIANGLES

Classification

By sides	By angles
Equilateral: 3 equal sides	Acute: all angles under 90°
Isosceles: 2 equal sides	Right: one 90° angle
Scalene: no equal sides	Obtuse: one angle over 90°

Key facts

- Angles add up to **180°** . Exterior angle = sum of the two remote interior angles.
- Triangle inequality: the sum of any two sides is greater than the third.
- The longest side is opposite the largest angle.

Congruence and similarity

Congruent (same size and shape)	Similar (same shape)
SSS, SAS, ASA, AAS, HL (right triangles only)	AA; SSS similarity (all 3 sides proportional); SAS similarity (2 sides proportional, included angle equal)

Side ratio $k \rightarrow$ perimeter ratio k , area ratio k^2 .

Quadrilateral comparison

Shape	Sides	Angles	Diagonals	Area
Parallelogram	Opposite sides parallel and equal	Opposite equal; consecutive supplementary	Bisect each other	base $\times$ height
Rectangle	Opposite sides parallel and equal	All 90°	Equal; bisect each other	$L \times W$
Rhombus	All sides equal	Opposite equal	Perpendicular; bisect each other and the angles	$\frac{1}{2} \times d_1 \times d_2$
Square	All sides equal	All 90°	Equal; perpendicular; bisect each other	s^2
Trapezoid	At least one pair of parallel sides (the bases)	Angles along each leg add to 180°	Equal only if isosceles	$\frac{1}{2}(b_1 + b_2) \times h$
Kite	Two pairs of adjacent sides equal	One pair of opposite angles equal	Perpendicular; one bisects the other	$\frac{1}{2} \times d_1 \times d_2$

Some courses define a trapezoid as having exactly one pair of parallel sides. Use your class's definition.

Common mistake: A square is both a rectangle and a rhombus, but not every rectangle is a square. Rectangle diagonals are equal; rhombus diagonals are perpendicular but not equal.

Special segments and centers

Segment	Meeting point	Notes
Median	Centroid	Splits each median 2:1 from the vertex
Altitude	Orthocenter	Perpendicular from vertex to opposite side
Angle bisector	Incenter	Center of the inscribed circle
Perp. bisector	Circumcenter	Center of the circumscribed circle

Area formulas

- Basic: $A = \frac{1}{2} \times \text{base} \times \text{height}$
- Two sides + included angle: $A = \frac{1}{2}ab \sin C$
- Equilateral: $A = (\sqrt{3}/4)s^2$
- Heron: $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = (a + b + c)/2$

Common mistake: AAA proves similarity, **not** congruence. SSA proves neither (except HL in right triangles).

3 RIGHT TRIANGLES AND TRIGONOMETRY

Pythagorean theorem: $a^2 + b^2 = c^2$ (c is the hypotenuse)

Common triples (and multiples): 3-4-5, 5-12-13, 8-15-17, 7-24-25

Special right triangles

Triangle	Side ratio
45° - 45° - 90°	$x : x : x\sqrt{2}$
30° - 60° - 90°	$x : x\sqrt{3} : 2x$ (short leg is opposite 30°)

SOH-CAH-TOA (Opp = opposite, Adj = adjacent, Hyp = hypotenuse)

$\sin \theta = \text{Opp} / \text{Hyp}$	$\cos \theta = \text{Adj} / \text{Hyp}$	$\tan \theta = \text{Opp} / \text{Adj}$
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Exact values

θ	30°	45°	60°
sin	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
cos	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$
tan	$\sqrt{3}/3$	1	$\sqrt{3}$

Any triangle

- Law of sines: $a/\sin A = b/\sin B = c/\sin C$
- Law of cosines: $c^2 = a^2 + b^2 - 2ab \cos C$

Common mistake: "Opposite" and "adjacent" depend on which angle you use, but the hypotenuse never changes. Check your calculator is in **degree mode**.

4 POLYGONS AND QUADRILATERALS

Polygon formulas (n = number of sides)

Quantity	Formula
Sum of interior angles	$(n - 2) \times 180^\circ$
Each interior angle (regular)	$(n - 2) \times 180^\circ / n$
Sum of exterior angles	360°
Each exterior angle (regular)	$360^\circ / n$
Number of diagonals	$n(n - 3) / 2$

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5 CIRCLES

Parts: radius, diameter ($2r$), chord, secant (line that intersects a circle at two points), tangent (touches at one point), arc, sector, segment

Formulas

Quantity	Formula
Circumference	$C = 2\pi r = \pi d$
Area	$A = \pi r^2$
Arc length	$(\theta/360^\circ) \times 2\pi r$
Sector area	$(\theta/360^\circ) \times \pi r^2$

Angle rules

Angle	Equals
Central angle	Its intercepted arc
Inscribed angle	$\frac{1}{2}$ its intercepted arc
Angle in a semicircle	90°
Angle formed by a tangent and a chord	$\frac{1}{2}$ its intercepted arc
Two chords crossing inside	$\frac{1}{2}$ (sum of the two arcs)
Two secants/tangents meeting outside	$\frac{1}{2}$ (far arc – near arc)

Theorems

- A tangent is perpendicular to the radius at the point of tangency.
- Two tangents from the same outside point are equal in length.
- A perpendicular from the center to a chord bisects the chord.
- Opposite angles of a cyclic quadrilateral add to 180° .

Segment lengths

- Intersecting chords: $PA \times PB = PC \times PD$ (the two pieces of one chord multiplied equal the two pieces of the other)
- Two secants from one outside point: (whole secant) $\times$ (outside part) = (whole secant) $\times$ (outside part)
- Tangent and secant: $\text{tangent}^2 = (\text{whole secant}) \times (\text{outside part})$

Common mistake: A central angle equals its arc, but an inscribed angle is **half** its arc. Use the radius, not the diameter, in area formulas.

6 COORDINATE GEOMETRY

Concept	Formula
Distance	$d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$
Midpoint	$((x_1 + x_2)/2, (y_1 + y_2)/2)$
Slope	$m = (y_2 - y_1)/(x_2 - x_1)$
Parallel lines	Equal slopes
Perpendicular lines	Nonvertical slopes are negative reciprocals
Horizontal / vertical line	Slope 0 / slope undefined

Equations of a line

Form	Equation
Slope-intercept	$y = mx + b$
Point-slope	$y - y_1 = m(x - x_1)$
Standard	$Ax + By = C$

Equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$, center (h, k) , radius r

Common mistake: The signs in a circle equation are flipped: $(x + 3)^2$ means $h = -3$. The right side is r^2 , not r . In the slope formula, subtract the coordinates in the same order on top and bottom.

7 AREA, SURFACE AREA, AND VOLUME

2D area

Shape	Area	Shape	Area
Square	s^2	Trapezoid	$\frac{1}{2}(b_1 + b_2) \times h$
Rectangle	$L \times W$	Rhombus / Kite	$\frac{1}{2} \times d_1 \times d_2$
Parallelogram	$b \times h$	Circle	πr^2
Triangle	$\frac{1}{2} \times b \times h$	Regular polygon	$\frac{1}{2} \times \text{apothem} \times \text{perimeter}$

3D solids

L = length, W = width, H = height, B = base area, P = base perimeter, ℓ = slant height. All solids are right solids.

Solid	Lateral area	Total surface area	Volume
Cube	$4s^2$	$6s^2$	s^3
Rectangular prism	$2H(L + W)$	$2(LW + LH + WH)$	$L \times W \times H$
Right prism	$P \times H$	$P \times H + 2B$	$B \times H$
Cylinder	$2\pi rH$	$2\pi r^2 + 2\pi rH$	$\pi r^2 H$
Regular pyramid	$\frac{1}{2} \times P \times \ell$	$B + \frac{1}{2} \times P \times \ell$	$\frac{1}{3} \times B \times H$
Cone	$\pi r\ell$	$\pi r^2 + \pi r\ell$	$\frac{1}{3}\pi r^2 H$

Sphere: surface area = $4\pi r^2$, volume = $(4/3)\pi r^3$

Scale factor k : lengths $\times k$, areas $\times k^2$, volumes $\times k^3$

Common mistake: Slant height (ℓ) is not the same as height (H). For a cone, $\ell^2 = r^2 + H^2$. Don't forget the $\frac{1}{3}$ in cone and pyramid volumes.

8 TRANSFORMATIONS

Transformation	Does	Rule
Translation	Slides	$(x, y) \rightarrow (x + a, y + b)$
Reflection over x-axis	Flips	$(x, y) \rightarrow (x, -y)$
Reflection over y-axis	Flips	$(x, y) \rightarrow (-x, y)$
Reflection over $y = x$	Flips	$(x, y) \rightarrow (y, x)$
Reflection over $y = -x$	Flips	$(x, y) \rightarrow (-y, -x)$
Rotation 90° counterclockwise	Turns	$(x, y) \rightarrow (-y, x)$
Rotation 180°	Turns	$(x, y) \rightarrow (-x, -y)$
Rotation 270° counterclockwise	Turns	$(x, y) \rightarrow (y, -x)$
Dilation (center at origin)	Resizes	$(x, y) \rightarrow (kx, ky)$

Dilation: $k > 1$ enlarges, $0 < k < 1$ shrinks.

Isometries (preserve size and shape): translations, reflections, rotations. A dilation with $k \neq 1$ changes size, so the image is similar, not congruent.

Common mistake: A 90° clockwise rotation is the same as 270° counterclockwise. Check the direction before applying the rule.

9 PROOF TOOLKIT

Reasons bank

Reason	When to use it
Given	Information stated in the problem
Definition of midpoint	Splits a segment into two equal parts
Definition of angle bisector	Splits an angle into two equal angles
Definition of perpendicular	Lines intersect to form right angles
Reflexive property	A side or angle equals itself (shared side)
Symmetric property	If $a = b$, then $b = a$
Transitive property	If $a = b$ and $b = c$, then $a = c$
Substitution	Replace a quantity with an equal one

Reason	When to use it
Addition / Subtraction property	Add or subtract equal amounts from equal things
Vertical angles theorem	Vertical angles are equal
Linear pair postulate	Linear pair angles are supplementary
Parallel line theorems	Corresponding, alternate interior, and co-interior angles
Triangle angle sum	Angles add to 180°
Isosceles triangle theorem	Equal sides have equal opposite angles, and vice versa
SSS, SAS, ASA, AAS, HL	Prove triangles congruent
CPCTC	Corresponding parts of congruent triangles are congruent

Two-column proof template

Statements	Reasons
1. (what you know)	1. Given
2. (next fact)	2. (definition, property, or theorem)
3. $\triangle ABC \cong \triangle DEF$	3. (SSS, SAS, ASA, AAS, or HL)
4. (what you want to show)	4. CPCTC

Quick strategy: Mark the given information on the diagram, write down what you need to prove, then work backward to find which triangles or angles will get you there.

Common mistake: Using CPCTC before proving the triangles congruent. Never use the statement you are trying to prove as one of your reasons.